CHAPTER 3

ECONOMIC EVALUATION OF ALTERNATIVES

Facility investment decisions represent major commitments of corporate resources and have serious consequences on the profitability and financial stability of a corporation. It is important to evaluate facilities rationally with regard to both the economic feasibility of individual projects and the relative net benefits of alternative and mutually exclusive projects.

3.1 Economic Evaluation

Engineering economic analysis is more than simply solving interest problems. The decision process requires that the outcomes of feasible alternatives be arranged so that they may be judged for economic efficiency in terms of the solution criteria.

A systematic approach for economic evaluation of facilities consists of the following steps:

1. Generate a set of projects or purchases for investment consideration.
2. Establish the planning horizon for economic analysis.
3. Estimate the cash flow profile for each project.
4. Specify the minimum attractive rate of return (MARR).
5. Establish the criterion for accepting or rejecting a proposal, or for selecting the best among a group of mutually exclusive proposals, on the basis of the objective of the investment.
6. Perform sensitivity or uncertainty analysis.
7. Accept or reject a proposal on the basis of the established criterion.
The basic principle in assessing the economic costs and benefits of new facility investments is to find the aggregate of individual changes in the welfare of all parties affected by the proposed projects. The changes in welfare are generally measured in monetary terms, but there are exceptions, since some effects cannot be measured directly by cash receipts and disbursements. Examples include the value of human lives saved through safety improvements or the cost of environmental degradation. To obtain an accurate estimate of costs in the cash flow profile for the acquisition and operation of a project, it is necessary to specify the resources required to construct and operate the proposed physical facility, given the available technology and operating policy.

Equivalence provides the logic by which we may adjust the cash flow for a given alternative into some equivalent or series. To apply the selection criterion, we must resolve them into comparable units. There are many methods to compare alternatives and finding equivalent consequences:

- Present worth analysis.
- Equivalent uniform annual cost analysis.
- Internal rate of return method.
- Benefit/cost ratio method.
- Payback period.

### 3.2 Planning Horizon

When comparing different alternatives for economic evaluation, each alternative has its own technical and financial properties. Also, each alternative has its own service life. Accordingly, in this analysis, careful consideration must be given to the time period covered by the analysis. The task to be accomplished, usually, has a time period associated with it. In this case, the consequences of each alternative must be considered for this period of time which is called the analysis period or the planning horizon.

The analysis period should be determined from the situation. In some analysis with rapidly changing technologies, a short analysis period or planning horizon may be
selected. Industries with more stable technologies might use a longer period, while government agencies frequently use analysis periods extending to fifty years or more. In civil engineering projects, the analysis period may be considered infinity like bridges, etc.

There are three different analysis-period situations in economic analysis problems:
- The alternatives have equal useful life or analysis period.
- The alternatives have different useful lives or analysis period.
- There is an infinite analysis period.

In the case of equal lives, it just needed to do the comparison based on the method used such as the present value, etc. In case of different lives, each alternative should be considered for its whole life period. The variation in the useful lives of the different alternatives means we no longer have a situation for fixed output. Therefore, it is important to select an analysis period for all alternatives and judge the consequences of all alternatives during the selected analysis period.

In the case of different analysis periods, one method is to select an analysis period which is the least common multiple of their useful lives. For example, when comparing two alternatives with 10 and 5 years lives, we may select the analysis period as 10 years. Thus means that the shorter life alternative is considered to be replaced with the same specification to last for another 5 years.

### 3.3 Life Cycle Costing

In the past, economic assessment of alternative designs, construction, or investments has been based on initial (first) cost which ignores the total cost incurred for the investment throughout its lifetime. The concept of life cycle costing provides an economic tool which takes into account total costs for an investment during its life span. Life cycle costing can be defined as “an economic assessment of alternative designs, construction, or other investments considering all significant costs of initial costs and ownership costs
over economic life of each alternative”. The life cycle costing analysis cannot be carried out without considering the following:

- Total costs
- Concept of the time value of money
- Initial costs: including the following: initial construction cost, design cost, land cost and finance cost
- Ownership costs: including operating costs (maintenance, repairs and utility bills).
- Replacement costs (cost of replacing the project after it runs its economic life).
- The concept of the time value of money. The value of money today is not equal to the same amount of money in the past or in the future. This concept considers the following: initial costs \((P)\) (present value), discount or interest rate \((i\%)\), life time of an investment \((n)\).
- Two main methods can be used: present worth method and uniform annual method.

### 3.4 Present Worth Analysis

Present worth is one of the ways to compare alternatives. It is most frequently used to determine the present value of future money receipts and disbursements. In the future, income and costs are known, and then using a suitable interest rate, the present worth can be calculated. In present worth analysis, careful consideration must be given to the time period covered by the analysis. Usually, the task to be accomplished has a time period associated with it. Accordingly, the analysis of each alternative must be considered for this period of time, which is named as described before the analysis period of the planning horizon.

In present worth analysis, the alternative with the maximum present worth \((PW)\) of benefits minus present worth of cost is always selected. This criterion is called the *net present worth criterion* \((NPW)\). As such,

\[
NPW = PW \text{ of benefits} - PW \text{ of costs}
\]  

(3.1)
3.4.1 Present Worth Comparison of Equal-Lived Alternatives

Example 3.1: Make a present-worth comparison of the equal-service life projects for which costs are shown below, if \( i = 10\% \). Which project would you select?

<table>
<thead>
<tr>
<th></th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, ( P )</td>
<td>LE 2,500,000</td>
<td>LE3,500,000</td>
</tr>
<tr>
<td>Annual operating cost, ( A )</td>
<td>LE 900,000</td>
<td>LE 700,000</td>
</tr>
<tr>
<td>Salvage value, ( F )</td>
<td>LE 200,000</td>
<td>LE 350,000</td>
</tr>
<tr>
<td>Project service life (years)</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Solution:

\[
P_A = LE2,500,000 + LE900,000 \times (P/A, 10\%, 5) - LE200,000 \times (P/F, 10\%, 5) \\
   = LE5,788,000 \\
\]

\[
P_B = LE3,500,000 + LE700,000 \times (P/A, 10\%, 5) - LE350,000 \times (P/F, 10\%, 5) \\
   = LE5,936,000 \\
\]

Project A should be selected since \( P_A < P_B \)

Example 3.2: Two machines are under consideration for a carpentry workshop. Machine A will have a first cost of LE5,000, an annual operation and maintenance cost of LE300, and a LE1,200 salvage value. Machine B will have a first cost of LE7,000, an annual operation and maintenance cost of LE200, and a LE1,300 salvage value. If both machines are expected to last for ten years, determine which machine should be selected on the basis of their present-worth values using an interest rate of 10%.

Solution:
<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost, ( P )</td>
<td>LE5,000</td>
<td>LE7,000</td>
</tr>
<tr>
<td>Operation and Maintenance cost, ( A )</td>
<td>LE300</td>
<td>LE200</td>
</tr>
<tr>
<td>Salvage value, ( F )</td>
<td>LE1200</td>
<td>LE1300</td>
</tr>
<tr>
<td>Service life (years), ( n )</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
P_A = LE5,000 + LE300 \left( \frac{P}{A}, 10\%, 10 \right) - LE1,200 \left( \frac{P}{F}, 10\%, 10 \right) = LE6,380.9 \\
P_B = LE7,000 + LE200 \left( \frac{P}{A}, 10\%, 10 \right) - LE1,300 \left( \frac{P}{F}, 10\%, 10 \right) = LE7,727.85
\]

Project A should be selected since \( P_A < P_B \)

**Example 3.3:** A truck, whose price is $18,600, is being paid for in 36 uniform monthly installments, including interest at 10 percent. After making 13 payments, the owner decides to pay off the remaining balance of the purchase price in one lump sum. How much is the lump sum?

**Solution:**

In problems like this the lump sum is the present worth of all the future (unpaid) payments. So, to solve the problem compute the payment and then compute the PW of the unpaid payments at the stated interest rate.

\[
A = 18,600 \left( \frac{A}{P}, 0.83\%, 36 \right) \\
A = 18,600 \left[ \frac{(0.00833(1 + .00833)^{36})/((1 + .00833)^{36} - 1)}{0.00833(1 + .00833)^{36}} \right] \\
A = LE600.22
\]

After 13 months: 36 - 13 = 23 payments remain

\[
P = 600.22 \left( \frac{P}{A}, 0.83\%, 23 \right) \\
P = 600.22 \left[ \frac{((1 + .00833)^{23} - 1)/(.00833(1 + .00833)^{23})}{0.00833} \right] \\
P = LE12,515.45
\]

**Example 3.4:** A firm is considering which of two mechanical devices to install to reduce costs in a particular situation. Both devices cost LE1000 and have useful lives of five years and no salvage value. Device A is expected to
result in LE300 savings annually. Device B will provide savings for LE400 the first year but will decline LE50 annually. With interest rate 7%, which device should the firm purchase?

Solution:

Both devices have the same useful lives and both costs LE1000. The appropriate decision criterion is to choose the alternative that maximizes the present worth of benefits. The cash flow diagram is shown below.

\[
PW_A = 300\left(\frac{P}{A}, 7\%, 5\right) = 300(4.100) = LE1230
\]

\[
PW_B = 400\left(\frac{P}{A}, 7\%, 5\right) - 50\left(\frac{P}{G}, 7\%, 5\right)
\]

\[
= 400(4.100) - 50(7.647) = LE1257.65
\]

Device B has the largest present worth of benefits, therefore, it the preferred alternative. It is worth noting that, if we ignored the time value of money, both alternatives provide LE1500 benefits over the five years. Device B provides more benefits in the first two years and smaller in the last two years, this more rapid flow of money from B, although the total value equal that of A, results in greater present worth of benefits.

3.4.2 Present Worth Comparison of Different-Lived Alternatives

In the previous examples, the useful life of each alternative was equal to the analysis period. But, there will be many situations where the alternatives have useful lives different from the analysis period. This situation will be examined in this section.
Example 3.5: A purchasing agent is considering the purchase of some new equipment for the mailroom. Two different manufacturers have provided quotations. An analysis of the quotations indicates the following:

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Cost</th>
<th>Useful life, years</th>
<th>End-of-useful-life salvage value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>LE1500</td>
<td>5</td>
<td>LE200</td>
</tr>
<tr>
<td>B</td>
<td>LE1600</td>
<td>10</td>
<td>EL325</td>
</tr>
</tbody>
</table>

Which manufacturer’s equipment should be selected? Assume 7% interest rate and equal maintenance cost.

Solution: As equipment A has a useful life of 5 years and B has useful life of 10 years, one method is to select an analysis period which is the least common multiplier of the lives of both equipment. Thus, we would compare the ten-year life of B against an initial purchase of A plus its replacement with a new one with the same values. The basis is to compare both alternatives on 10-years period.

\[
P_{WA} = LE1,500 + LE(1,500-200) \left(\frac{P}{F}, 7\%, 5\right) - LE200 \left(\frac{P}{F}, 7\%, 10\right) \\
= LE1,500 + 1300(0.713) - 200(0.5083) = LE2,325 \\
P_B = LE1,600 - LE325 \left(\frac{P}{F}, 7\%, 10\right) \\
= LE1,600 - 325(0.5083) = LE1,435
\]

Equipment B should be selected since \( PW_B < PW_A \)
Example 3.6: Make a present-worth comparison of the different-life machines for which costs are shown below, if $i = 15\%$. Which machine would you select?

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost, $P$</td>
<td>LE11,000</td>
<td>LE18,000</td>
</tr>
<tr>
<td>Operation and Maintenance cost, $A$</td>
<td>LE3,500</td>
<td>LE3,100</td>
</tr>
<tr>
<td>Salvage value, $F$</td>
<td>LE1,000</td>
<td>LE2,000</td>
</tr>
<tr>
<td>Service life (years), $n$</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Solution: The cash flow for one cycle of an alternative must be duplicated for the least common multiple of years, so that service life is compared over the total life for each alternative. Since the machines have different lives, they must be compared over the least common multiple of years, 18 years.

\[ P_A = LE11,000 + LE11,000 \left( P/F, 15\%, 6 \right) + LE11,000 \left( P/F, 15\%, 12 \right) + LE3,500 \left( P/A, 15\%, 18 \right) - LE1,000 \left( P/F, 15\%, 6 \right) - LE1,000 \left( P/F, 15\%, 12 \right) - LE1,000 \left( P/F, 15\%, 18 \right) = LE38,559 \]

\[ P_B = LE18,000 + LE18,000 \left( P/F, 15\%, 9 \right) + LE3,100 \left( P/A, 15\%, 18 \right) - LE2,000 \left( P/F, 15\%, 9 \right) - LE2,000 \left( P/F, 15\%, 18 \right) = LE41,384 \]

Machine A should be selected since $P_A < P_B$
Example 3.7: A company is trying to decide between two different garbage disposals. A regular (RS) disposal has an initial cost of LE65 and a life of 4 years. The alternative is a corrosion-resistant disposal constructed of stainless steel (SS). The initial cost of the SS disposal is LE110, but it is expected to last 10 years. The SS disposal is expected to cost LE5 per year more than the RS disposal. If the interest rate is 6%, which disposal should be selected, assuming both have no salvage value?

Solution: Since the disposals have different lives, they must be compared over the least common multiple of years, which is 20 years.

<table>
<thead>
<tr>
<th></th>
<th>RS</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>LE65</td>
<td>LE110</td>
</tr>
<tr>
<td>Additional cost per year</td>
<td>-</td>
<td>LE5</td>
</tr>
<tr>
<td>Salvage value, F</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Service life (years), n</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
P_{RS} = \text{LE65} + \text{LE65 (P/F, 6\%, 4)} + \text{LE65 (P/F, 6\%, 8)} + \text{LE65 (P/F, 6\%, 12)} + \text{LE65 (P/F, 6\%, 16)} = \text{LE215}
\]

\[
P_{SS} = \text{LE110} + \text{LE110 (P/F, 6\%, 10)} + \text{LE5 (P/A, 6\%, 20)} = \text{LE229}
\]

Disposal RS should be selected since \( P_{RS} < P_{SS} \)
3.4.3 Present Worth Comparison of Infinite Analysis Periods

In the present worth analysis, sometimes an infinite analysis period ($n = \infty$) is encountered. Alternatives such as roads, dams, bridges or whatever is sometimes considered permanent. In these situations a present worth analysis would have an infinite analysis period. This analysis is called capitalized cost. A capitalized cost is the present sum of money that would need to be set aside now, at some interest rate, to yield the funds required to provide the service indefinitely.

Example 3.8: A city plans a pipeline to transport water from a distant watershed area to the city. The pipeline will cost LE8 million and have an expected life of seventy years. The city anticipates it will need to keep the water line in service indefinitely. Compute the capitalized cost assuming 7% interest.

Solution: We have the capitalized cost equation $P = \frac{A}{i}$, which is simple to apply when there are end-of-period disbursements $A$. In this case, the LE8 million repeats every 70 years. We can find $A$ first based on a present LE8 million disbursement.

\[ A = P\left(\frac{A}{P}, i, n\right) = \text{LE}8,000,000(0.0706) = \text{LE}565,000 \]

Now, the infinite series payment formula could be applied for $n = \infty$:

Capitalized cost $P = A / i = 565,000 / 0.07 = \text{LE}8,071,000$

3.5 Equivalent Uniform Annual Worth Analysis

Instead of computing equivalent present sum, in this section alternatives could be compared based on their equivalent annual costs (cash flows). Based on particular situation, the equivalent uniform annual cost ($EUAC$), the equivalent uniform annual benefits ($EUAB$), or their difference could be calculated.

In chapter two, techniques to convert money, at one point in a time, to sum equivalent sum or series were presented. In this section, the goal is to convert money into an equivalent uniform annual cost or benefits. The major advantage of this method is that it is not necessary to make the comparison over the same number of years when the
alternatives have different lives. The reason for that, it is an equivalent annual cost over the life of the project.

*Example 3.9:* If the minimum required rate of return is 15% which project should be selected?

<table>
<thead>
<tr>
<th></th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost</td>
<td>LE26,000,000</td>
<td>LE36,000,000</td>
</tr>
<tr>
<td>Annual maintenance cost</td>
<td>LE800,000</td>
<td>LE300,000</td>
</tr>
<tr>
<td>Annual labor cost</td>
<td>LE11,000,000</td>
<td>LE7,000,000</td>
</tr>
<tr>
<td>Extra income taxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salvage value</td>
<td>LE2,000,000</td>
<td>LE3,000,000</td>
</tr>
<tr>
<td>Project service life (years)</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

*Solution:*

In this case, all costs or savings (cash flows) which occur during the life time of an investment are discounted to a uniform annual series of cash flows over the life time of the investment.

\[
EUAC_A = LE26,000,000(A/P, 15\%, 6) - LE2,000,000(A/F, 15\%, 6) + \]
\[
LE11,800,000 = LE18,442,000
\]
\[ EUAC_B = \text{LE}36,000,000(A/P, 15\%, 10) - \text{LE}3,000,000 (A/F, 15\%, 10) + \text{LE}9,900,000 = \text{LE}16,925,000 \]

Project B should be selected since \( EUAC_B < EUAC_A \)

**Example 3.10:** An asset depreciates uniformly from a first cost of \( \text{LE}50,000 \) to zero over a 20-year time frame. If operating costs are initially \( \text{LE}1,500 \) but increase by \( \text{LE}2,000 \) per year and revenues are \( \text{LE}20,000 \) per year but decrease by \( \text{LE}1,000 \) per year what is the EAW if the machine is replaced every 10 years and the interest rate is 5%.

**Solution:**
\[
EAUC = -(50,000 - 25,000)(A/P, 5, 10) - 1,500 - 2,000(A/G, 5, 10) \
+ 20,000 - 1,000(A/G, 5, 10) \
= -25,000(0.1295) - 1,500 - 2,000(4.09909) + 20,000 \
- 1,000(4.09909) = \text{LE}2,965
\]

**Example 3.11:** In the construction of an aqueduct to expand the water supply of a city, there are two alternatives. Either a tunnel through a mountain or a pipeline can be laid on the ground. Given the following information which alternative should be adopted? Assume a 6% interest rate.

<table>
<thead>
<tr>
<th></th>
<th>Tunnel</th>
<th>Pipeline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>LE5,500,000</td>
<td>LE5,000,000</td>
</tr>
<tr>
<td>Annual maintenance</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Salvage value</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Project service life (years)</td>
<td>Permanent</td>
<td>50</td>
</tr>
</tbody>
</table>

**Solution:** For the tunnel with its permanent useful life, \( n \) considered to be infinity.
\[
EAUC_{tunnel} = Pi = 5,500,000 (0.06) = \text{LE}330,000
\]
For the pipeline,
\[
EAUC_{pipeline} = 5,000,000(A/P, 6, 50) = \text{LE}317,000
\]
Then, select the pipeline option.
3.6 The Rate of Return Method

Currently we use three different methods of determining whether an economic proposal is valid. Present worth comparisons, Future worth comparisons and Equivalent annual worth comparisons. While these methods will tell us which alternative is the best, and whether we are gaining a greater return than our minimum attractive rate of return (MARR), we don’t know the rate of return that we are actually receiving. The objective of the rate of return method is to find the rate of return (i% percentage) for an investment over a specific service life. The rate of return method considers all the cash follows that occur during the life cycle of an investment. There are two methods to find the rate of return for an investment. These are: the present worth Method and the equivalent uniform annual cost (EUAC) method.

Internal rate of return (IRR) calculations (i) tell us the exact rate of return we are receiving on an individual investment. These calculations must be done through trial and error, or by using commercially available software. If projects being assessed are independent of each other then any project with an IRR greater than the MARR should be accepted. IRR is defined as the interest rate paid on the unpaid balance of a loan such that the payment schedule makes the unpaid loan balance equal to zero when the final payment is made.

For example, we might invest LE5000 in a machine with a five-year useful life and a EUAB of LE1252. Now, a question is raised: what rate of return would we receive on this investment? The least IRR is the one that makes the NPV of all payments equal to zero. The five payments of LE1252 are equivalent to a present sum of LE5000 when interest rate is 8%. Therefore, the IRR on this investment is 8% as shown in the following table.

\[ IRR \] is also defined as the investment rate that makes the PV of all expenditures equals the PV of all income; i.e., the NPV equals zero. To calculate the IRR, the NPV of all values are calculated as a function of the interest rate (i). Then, we calculate (i) that makes the NPV equals zero.
In present worth and equivalent uniform annual worth analysis, one must select an interest rate for use in the calculations and this may be difficult. In rate of return analysis, no interest rate is introduced into the calculations. Instead we compute an internal rate of return from the cash flow. To decide proceeding or not, the calculated rate of return is compared with a preselected minimum attractive rate of return.

### 3.6.1 IRR for One Alternative

When deciding on one alternative, then the alternative is acceptable if it brings a positive IRR or an IRR greater than the MARR.

**Rate of return calculation by the present worth method:**

The calculations are done in three steps:
- Draw a cash flow diagram.
- Set up the rate of return equation in the form:
  \[
  0 = \pm P + \sum_{j=1}^{n} F(P/F, i\%, n) \pm A(P/A, i\%, n)
  \]  
  \(\text{(3.2)}\)
- Select values of \(i\) by trial and error until the equation is balanced.

**Rate of Return Calculation by the EUAC Method:**

The calculations are done in three steps:
- Draw a cash flow diagram.
- Set up the rate of return equation in the form:
  \[
  0 = \pm P (A/P, i\%, n) \pm A
  \]  
  \(\text{(3.3)}\)
- Select values of \(i\) by trial and error until the equation is balanced.
Example 3.12: An investment resulted in the following cash flow. Compute the rate of return.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow</td>
<td>LE-700</td>
<td>+LE100</td>
<td>+LE175</td>
<td>+LE250</td>
<td>+LE325</td>
</tr>
</tbody>
</table>

Solution: \( EUAB - EUAC = 0 \)

\[
100 + 75(A/G, i, 4) - 700(A/P, i, 4) = 0
\]

This equation is solved using trial and error. Try \( i = 5\% \)

\[
100 + 75(A/G, 5, 4) - 700(A/P, 5, 4) = +LE11
\]

Try \( i = 8\% \)

\[
100 + 75(A/G, 8, 4) - 700(A/P, 8, 4) = -LE6
\]

\( i \) is between 5 and 8\%. Then try another value for \( i \) or find \( i \) by interpolation, then \( i = 7\% \).

Example 3.13: If LE5,000 is invested now, this is expected to yield LE100 per year for 10 years and LE7,000 at the end of 10 years, what is the rate of return?

Solution:

Using the equation:

\[
0 = - P + \sum_{j=1}^{n} F (P/F, i\%, n) + A (P/A, i\%, n)
\]

0 = - 5,000 + 100 \( (P/A, i\%, 10) \) + 7,000 \( (P/F, i\%, 10) \)

Try \( i = 5\% \):

\[
0 = - 5,000 + 100 \times (P/A, 5\%, 10) + 7,000 \times (P/F, 5\%, 10)
\]

\( = +LE69.46 \)

Try \( i = 6\% \):

\[
0 = - 5,000 + 100 \times (P/A, 6\%, 10) + 7,000 \times (P/F, 6\%, 10)
\]

\( = -LE355.19 \)

By interpolation \( i = 5.16\% \)
Example 3.14: Solve the problem presented in example 3.13 using the EUAW.

Solution: Using the equation: 
\[ 0 = \pm P \left( \frac{A}{P}, i\%, n \right) \pm A \]

\[ 0 = -5,000 \left( \frac{A}{P}, i\%, 10 \right) + 7,000 \left( \frac{A}{F}, i\%, 10 \right) + 100 \]

Try \( i = 5\% \):
\[ 0 = -5,000 \left( \frac{A}{P}, 5\%, 10 \right) + 7,000 \left( \frac{A}{F}, 5\%, 10 \right) + 100 \]
\[ = + \text{LE} 9.02 \]

Try \( i = 6\% \):
\[ 0 = -5,000 \left( \frac{A}{P}, 6\%, 10 \right) + 7,000 \left( \frac{A}{F}, 6\%, 10 \right) + 100 \]
\[ = - \text{LE} 48.26 \]

By interpolation: \( i = 5.16\% \)

3.6.2 Comparing Two Alternative Using the IRR Method

When two alternatives are compared, the IRR for each one is calculate and the alternative with the highest IRR will be chosen given that it satisfies the MARR. If the cash flows for one or all alternatives contain expenditures only, in this case we couldn’t compute the IRR and accordingly we couldn’t compare these alternatives using the IRR. Do not simply pick the project with the highest rate of return value. In this case the rate of return analysis is performed by computing the incremental rate of return (\( \Delta IRR \)) on the difference between the alternatives. The cash flow for the difference between the alternatives is computed by taking the higher initial cost alternative minus the lower initial cost alternative. If the \( \Delta IRR \) is \( \geq \) the MARR then choose the higher cost alternative. If the \( \Delta IRR \) is \( \leq \) the MARR then choose the lower cost alternative.

Example 3.15: You are given the choice of selecting one of two alternatives. The cash flows of the alternatives as shown in the following table. If the MARR is 6\%, which one you select?

<table>
<thead>
<tr>
<th>Year</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-LE10</td>
<td>-LE20</td>
</tr>
<tr>
<td>1</td>
<td>+15</td>
<td>+28</td>
</tr>
</tbody>
</table>
Solution: Normally, we select the lesser-cost alternatives (alternative 1), unless we find the additional cost of alternative 2 produces sufficient additional benefits that we would prefer. So, we will evaluate the difference project.

<table>
<thead>
<tr>
<th>Year</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
<th>Alt. 2 – Alt. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-LE10</td>
<td>-LE20</td>
<td>-20 – (-10) = -10</td>
</tr>
<tr>
<td>1</td>
<td>+15</td>
<td>+28</td>
<td>28 – 15 = +13</td>
</tr>
</tbody>
</table>

\[ PW_{\text{cost}} = PW_{\text{benefit}} \]

\[ 10 = 13(P/F, i\%, 1) \]

\( (P/F, i\%, 1) = 10/13 = 0.7692; \text{ then } i = 30 \)

Thus means that the additional LE10 invested to obtain alternative 2 is superior to invest the LE10 elsewhere at 6% (MARR).

Example 3.16: The following information showing the cash flows for two alternatives. If the MARR is 15%, which one is preferred?

<table>
<thead>
<tr>
<th></th>
<th>Alternative 1</th>
<th>Alternative 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>LE8000</td>
<td>LE13000</td>
</tr>
<tr>
<td>Annual costs</td>
<td>3500</td>
<td>1600</td>
</tr>
<tr>
<td>Salvage value</td>
<td>-</td>
<td>2000</td>
</tr>
<tr>
<td>Useful life</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Solution: As all cash flows are expenditures, we use the incremental rate of return. Also, as the useful lives are different, we use the common multiplier for both alternative, 10 years for comparison. The cash flow for the two alternatives and the difference is shown in the next table.
<table>
<thead>
<tr>
<th>Year</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
<th>Alt. 2 – Alt. 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-8000</td>
<td>-13000</td>
<td>-5000</td>
</tr>
<tr>
<td>1-5</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>+2000</td>
<td>-11000</td>
</tr>
<tr>
<td>6-10</td>
<td>-3500</td>
<td>-1600</td>
<td>1900</td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>+2000</td>
<td>2000</td>
</tr>
</tbody>
</table>

\[
NPV_{2,1} = 0 = -5000 + 1900(P/A, i\%, 10) - 11000(P/F, i\%, 5) + 2000(P/F, i\%, 10)
\]

Solving by trial and error; then \( IRR_{2,1} = 12.65\% \)

Where \( IRR_{2,1} = \) is less than the MARR, then choose alternative 1.

*Note: in case of comparing more than two alternatives, to properly calculate an incremental IRR, follow the following steps:*

- List the projects in order of increasing capital.
- Check that the first project has an IRR greater than the MARR
- If it does it becomes the defender, if not reject it and try remaining projects in order until one is greater than the MARR
- Subtract the defender’s values (first cost, annual cost, annual revenues) from the challenger and calculate the IRR of the remainder
- If the IRR is greater than the MARR accept the challenger as the new defender, if not reject it and compare the next alternative
- Continue until all alternatives have been considered

*Example 3.17:* A small company is looking at expanding its business by purchasing a small new store that will operate for 10 years before being sold and replaced with a newer larger store. Three sites have been recommended to the owner each with different costs and expected revenues based on its
location. The company operates with a MARR of 15% before taxes. Rate the alternatives based on a) *PW* comparison and b) *IRR* comparison.

<table>
<thead>
<tr>
<th></th>
<th>SITE 1</th>
<th>SITE 2</th>
<th>SITE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land Purchase Price</td>
<td>100,000</td>
<td>150,000</td>
<td>160,000</td>
</tr>
<tr>
<td>Renovations</td>
<td>40,000</td>
<td>40,000</td>
<td>60,000</td>
</tr>
<tr>
<td>Resale</td>
<td>125,000</td>
<td>155,000</td>
<td>175,000</td>
</tr>
<tr>
<td>Expected Revenue</td>
<td>125,000</td>
<td>195,000</td>
<td>300,000</td>
</tr>
<tr>
<td>Annual Power Costs</td>
<td>35,000</td>
<td>55,000</td>
<td>75,000</td>
</tr>
<tr>
<td>Annual O &amp; M cost</td>
<td>66,000</td>
<td>109,000</td>
<td>184,000</td>
</tr>
</tbody>
</table>

**Solution:** First, determine total initial cost, net revenues and rank the sites in order of first cost.

<table>
<thead>
<tr>
<th></th>
<th>SITE 1</th>
<th>SITE 2</th>
<th>SITE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial costs</td>
<td>140,000</td>
<td>190,000</td>
<td>220,000</td>
</tr>
<tr>
<td>Resale</td>
<td>125,000</td>
<td>155,000</td>
<td>175,000</td>
</tr>
<tr>
<td>Net Revenues/year</td>
<td>24,000</td>
<td>31,000</td>
<td>41,000</td>
</tr>
</tbody>
</table>

a) **Present Worth Comparison where i =15%**

Site 1: \( PW = -140,000 + 125,000(P/F, 15, 10) + 24,000(P/A, 15, 10) \)
\[= -140,000 + 30,898 + 120,450 = LE11,348 \]

Site 2: \( PW = -190,000 + 155,000(P/F, 15, 10) + 31,000(P/A, 15, 10) \)
\[= -190,000 + 38,313 + 155,582 = LE3,895 \]

Site 3: \( PW = -220,000 + 175,000(P/F, 15, 10) + 41,000(P/A, 15, 10) \)
\[= -220,000 + 43,257 + 205,770 = LE29,027 \]

Therefore select Site 3 then Site 1 and then Site 2

b) **IRR Comparison**

Site 1: \(-140,000 + 125,000(P/F, i, 10) + 24,000(P/A, i, 10) = 0 \)

By trial and error \( i = 16.66\% \), Select Site 1 since \( IRR > MARR \)
Incremental site 2- site 1
\[-50,000 + 30,000(P/F, i, 10) + 7,000(P/A, i, 10) = 0\]
By trial and error \(i = 11.69\%\)
Select Site 1 since \(\Delta IRR < MARR\)

Incremental site 3- site 1
\[-80,000 + 50,000(P/F, i, 10) + 17,000(P/A, i, 10) = 0\]
By trial and error \(i = 19.79\%\)
Select Site 3 since \(\Delta IRR < MARR\)

Therefore select Site 3 then Site 1 and then Site 2; same as of the previous result.

3.7 Benefit/Cost Ratio Method

The Benefit/Cost (\(B/C\)) method is based on the ratio of the annual benefits to the annual costs for a particular project. It is used to compare between investment options based on a range of benefits, disbenefits, and costs to the owner. Benefits are advantages, expressed in terms of a monetary value to the owner (\(i.e.\) dollars, etc.). Disbenefits are disadvantages, expressed in terms of a monetary value to the owner (\(i.e.\) dollars, etc.). Costs are anticipated expenditures for construction, operation and maintenance, etc. the following Table shows examples of benefits, disbenefits, and costs:

<table>
<thead>
<tr>
<th>Item</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure of 11 million dollars for a new highway</td>
<td>Cost</td>
</tr>
<tr>
<td>$100,000 annual income to local residents from tourists due to the construction of new highway</td>
<td>Benefit</td>
</tr>
<tr>
<td>$150,000 annual upkeep of highway</td>
<td>Cost</td>
</tr>
<tr>
<td>$250,000 annual loss to farmer due to loss of highway right-of-way</td>
<td>Disbenefit</td>
</tr>
</tbody>
</table>

The formula used for benefit to cost ratio analysis is:

\[
B/C = \frac{\text{Benefits} - \text{Disbenefits}}{\text{Costs}}
\]  

\[(3.4)\]
If the $B/C$ ratio is $\geq 1.0$, this means that the extra benefit(s) of the higher cost alternative justify the higher cost. If the $B/C$ ratio is $< 1.0$, this means that the extra cost is not justified and the lower cost alternative is selected. An alternative method that can be used to compare between projects, is to subtract the costs from the benefits that is $(B - C)$. If $(B - C)$ is $\geq 0$, this means that the project is acceptable. If $(B - C)$ ratio is $< 0$, this means that the project is rejected.

Rather than solving problems using present worth or annual cash flows analysis, we can base the calculations on the benefit-cost ratio, $B/C$. The $B/C$ is the ration of benefits to costs, or:

$$\frac{B}{C} = \frac{PW \text{ of benefit}}{PW \text{ of costs}} = \frac{EUAB}{EUAC} \geq 1 \quad (3.5)$$

**Example 3.18:** Two routes are considered for a new highway, Road A, costing LE4,000,000 to build, will provide annual benefits of LE750,000 to local businesses. Road B would cost LE6,000,000 but will provide EL700,000 in benefits. The annual cost of maintenance is LE300,000 for Road A and LE320,000 for Road B. If the service life of Road A is 20 years, and for Road B is 30 years, which alternative should be selected if the interest rate is 8%?

**Solution:** Tabulate the given data:

<table>
<thead>
<tr>
<th>Given Data</th>
<th>Road A</th>
<th>Road B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>LE4,000,000</td>
<td>LE 6,000,000</td>
</tr>
<tr>
<td>Annual Benefits</td>
<td>LE 750,000</td>
<td>LE 700,000</td>
</tr>
<tr>
<td>Annual Maintenance Cost</td>
<td>LE 300,000</td>
<td>LE 320,000</td>
</tr>
<tr>
<td>Service Life</td>
<td>20 years</td>
<td>30 years</td>
</tr>
</tbody>
</table>

$$B/C = (\text{Benefits} - \text{Disbenefits}) / \text{Costs}$$

$EUAC$ for Road A = $4,000,000 \times (A/P, 8\%, 20) + 300,000$

$$= 4,000,000 \times (0.10185) + 300,000 = \text{LE707,000}$$

$EUAC$ for Road B = $6,000,000 \times (A/P, 8\%, 30) + 320,000$

$$= 6,000,000 \times (0.08883) + 320,000 = \text{LE552,980}$$
\[ B/C \text{ for Road A} = \frac{750,000}{707,000} = 1.06 \]

\[ B/C \text{ for Road B} = \frac{700,000}{552,980} = 1.26 \]

Choose Road B

*Example 3.19:* Solve example 3.4 again using the \( B/C \) ration.

*Solution:*

The cash flow of both alternatives is shown in the figure below.

![Cash Flow Diagram](image)

**Device A:**

\[ PW_{\text{cost}} = \text{LE}1000 \]

\[ PW_{\text{benefit}} = 300(P/A, 7\%, 5) = 300(4.100) = \text{LE}1230 \]

Then \( B/C = 1230/1000 = 1.23 \)

**Device B:**

\[ PW_{\text{cost}} = \text{LE}1000 \]

\[ PW_{\text{benefit}} = 400(P/A, 7\%, 5) - 50(P/G, 7\%, 5) = 400(4.100) - 50(7.647) = \text{LE}1258 \]

Then \( B/C = 1258/1000 = 1.26 \)

To maximize the \( B/C \) ratio, choose device B.

*Example 3.20:* Two machines are being considered for purchase. If the interest rate is 10\%, which machine should be bought?
Solution: The solution is based on 12 years analysis period, so machine X will be replaced with same cash flows for identical periods. The cash flows are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Machine X</th>
<th>Machine Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>LE200</td>
<td>LE700</td>
</tr>
<tr>
<td>Annual benefits</td>
<td>95</td>
<td>120</td>
</tr>
<tr>
<td>Salvage value</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>Useful life, years</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

Machine X:

\[ EUAC = 200(A/P, 10\%, 6) - 50(A/F, 10\%, 6) \]
\[ = 200(0.2296) - 50(0.1296) = LE40 \]
\[ EUAB = LE95 \]

Machine Y:

\[ EUAC = 700(A/P, 10\%, 12) - 150(A/F, 10\%, 12) \]
\[ = 700(0.1468) - 150(0.0468) = LE96 \]
\[ EUAB = LE120 \]

Using the same way as we did when using the IRR analysis, calculate the incremental benefit cost ratio, \( \Delta B/\Delta C \) (machine Y – machine X)

\[ \Delta B/\Delta C = (120-95) / (96-40) = 0.45 \]

Since the incremental benefit cost ratio is less than 1, it represents an undesirable increment of investment. We therefore choose the lower cost alternative – machine X.

Note: in case of comparing more than two alternatives, follow the same procedure for the IRR. This will be used if only one alternative would be selected not more than one at the same time:
- List the projects in order of increasing capital.
- Calculate $B/C$ for the first alternative, if $B/C$ is greater than one, thus means that this alternative is acceptable and can be compared with other alternatives. If $B/C$ is less than one, then continue with other alternative until you reach an alternative with $B/C$ greater than 1.

- Compare the selected alternative from the previous step with the next alternative. Subtract $B/C$ for the selected with the next alternative (difference project). If $B/C$ for the difference project is greater than 1, then select the project with highest cost.

- Continue until all alternatives have been considered.

*Example 3.21:* A company has decided to build a factory on a particular site. There are two mutually exclusive proposals that have been developed for the main factory. There are also three secondary proposals for the main project. The present worth of the benefits and costs are shown below. Which combinations of projects are best if the company can only spend LE400,000?

<table>
<thead>
<tr>
<th>Main Proposals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Secondary Proposals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

*Solution:* All the possible combinations with cost less that LE400,000 are ranked in order in the following table:
<table>
<thead>
<tr>
<th>Combination</th>
<th>Benefits</th>
<th>Costs</th>
<th>B-C</th>
<th>B/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (A, 1, 2)</td>
<td>515,000</td>
<td>300,000</td>
<td>215,000</td>
<td>1.72</td>
</tr>
<tr>
<td>II (A, 3)</td>
<td>600,000</td>
<td>300,000</td>
<td>300,000</td>
<td>2.0</td>
</tr>
<tr>
<td>III (A, 1, 3)</td>
<td>675,000</td>
<td>350,000</td>
<td>325,000</td>
<td>1.93</td>
</tr>
<tr>
<td>IV (A, 2, 3)</td>
<td>740,000</td>
<td>400,000</td>
<td>340,000</td>
<td>1.85</td>
</tr>
<tr>
<td>V (B, 1, 2)</td>
<td>665,000</td>
<td>400,000</td>
<td>265,000</td>
<td>1.66</td>
</tr>
<tr>
<td>VI (B, 3)</td>
<td>750,000</td>
<td>400,000</td>
<td>350,000</td>
<td>1.87</td>
</tr>
</tbody>
</table>

Remember incremental comparison will give the correct result if B/C ratio method is adopted. Since all B/C ratios are greater than one, then we will compare all alternatives.

Since alternatives I and II have the same costs while benefits of alternative II is greater, then choose II.

**III-II**

\[
\Delta B/\Delta C = (675,000 - 600,000)/(350,000 - 300,000) = 1.333
\]

Accept III

**IV-III**

\[
\Delta B/\Delta C = (740,000 - 675,000)/(400,000 - 350,000) = 1.3
\]

Accept IV

Since alternatives V and IV have the same costs while benefits of alternative IV is greater, then choose IV.

**VI-IV**

Since both alternatives VI and IV have the same costs while benefit of alternative VI is greater, then Accept VI.
3.8 Payback Period

Payback period is defined as “the period of time required for the profit or other benefits from an investment to equal the cost of the investment”. The criterion in all situations is to minimize the payback period. Accordingly, this method concerns with determining the number of years or months (the time) required to recover all the invested money. When comparing between alternatives using this method, the alternative with the least payback period is selected.

Example 3.22: A company has decided to buy new equipment for a project with 4-year duration. There are two different equipment can be used for this project. The cash flows of those types are shown in the table below. What is the payback period for each equipment and which one should be selected? (Note that negative cash flow means expenditures).

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flows</th>
<th>Equipment A</th>
<th>Equipment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-LE35,000</td>
<td>-LE35,000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20,000</td>
<td>10,000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>10,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>15,000</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10,000</td>
<td>20,000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Cumulative cash flows</th>
<th>Equipment A</th>
<th>Equipment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-LE35,000</td>
<td>-LE35,000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-15,000</td>
<td>-25,000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-15,000</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10,000</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20,000</td>
<td>20,000</td>
<td></td>
</tr>
</tbody>
</table>

Solution: To find the payback period, find the time at which the cumulative cash flows equal zero.
As shown from the table above, the payback period for equipment A is 2 years, while it is 3 years for equipment B. It is worth noted that the investment in both alternatives is the same (LE35,000) and the net return is also the same (LE55,000). Accordingly, equipment A will be selected.

The payback as an economic evaluation method has the following advantages:
- Easy to use.
- Uses all the available cash flow not other information.
- Reduce the possibility of risk on projects as it selects the project with the least payback period.
- Well known economic evaluation method.

Despite these advantages, there are some concerns on using the payback period method:
- It is an approximate, rather than an exact, economic analysis method.
- It does not consider the time-value of money.
- All economic consequences beyond the payback period are ignored.
- It is not suitable for long-life projects as the actual cash flows may differ than that was expected at the time of analysis.

Despite this, payback period still used as a valid economic evaluation methods. It can be used for initial screening of different alternatives.

As mentioned before, one of the disadvantages of the payback period is that it ignores the time value of money. This drawback could be overcome by considering the time value of money, by determining the $PW$ of both the costs and the revenues and equating them to find the payback period.

*Example 3.23:* Calculate the payback period for the following two alternatives shown in the next table if the investment rate is 15%. Which one do you recommend?
Solution: Let’s assume that the payback period is \( n \) years. So, it is required to find \( n \) where the \( PW \) of cost equals the \( PW \) of benefits. Let’s assume that the salvage value is constant and can be obtained at any time for the payback period (if any).

\[
P_{\text{cost}} = P_{\text{benefits}}
\]

**Alternative 1:**

\[
12000 = 3000(\dfrac{P}{A}, 15, n1)
\]

Then, \( (P/A, 15, n1) = 4 \)

From the interest table, \( n1 \) lie between year 6 and year 7.

By interpolation \( n1 = 6.6 \) years

Or by using the investment equations:

\[
12000 = 3000[(1.15^{n1} - 1)/(0.15)(1.15)]
\]

Assume \( n1 = 6 \)

\[
3000(3.784) = 11352 < 12000
\]

Assume \( n1 = 7 \)

\[
3000(4.160) = 12480 > 12000
\]

By interpolation, \( n1 = 6.6 \) years

Then, the payback period for this alternative is 6.6 (almost 7 years) which almost equals the useful life of this alternative. As the payback period (6.6 years) is almost equal the useful life (7 years), thus means that the equipment is just returning its investments.
Alternative 2:

\[ 8000 = 1500(P/A, 15, n2) \]

Then, \((P/A, 15, n2) = 5.33\)

From the interest table, \(n2\) lie between year 11 and year 12.

By interpolation \(n2 = 11.5\) years

Or by using the investment equations:

\[ 8000 = 1500[(1.15^n2 - 1)/(0.15)(1.15)] \]

Assume \(n2 = 10\)

\[ 3000(5.019) = 7528.5 < 8000 \]

Assume \(n2 = 11\)

\[ 3000(5.234) = 7851 > 12000 \]

Assume \(n2 = 12\)

\[ 3000(5.214) = 8131.5 > 12000 \]

By interpolation, \(n2 = 11.5\) years

Then, the payback period for this alternative is 11.5 (almost 12 years).

As the payback period (11.5 years) is less than the equipment useful life (15 years), thus means that the equipment is capable to return all the invested money before its life ends.

Based on these results, alternative 1 will be selected as it has a shorter payback period. There is return for the investment \((IRR = 0)\) in alternative 1. While the \(IRR\) for alternative 2 will be greater than zero as there is still some returns after the payback period. To overcome this problem, we may compare the payback period over the useful life of each alternative and select the smaller ratio. Applying this for the previous example:

\[ \frac{n1}{useful\ life\ 1} = \frac{6.6}{7} = 0.94 \]

\[ \frac{n2}{useful\ life\ 2} = \frac{11.5}{15} = 0.77 \]

Based on this comparison, select alternative 2.
3.9 Inflation

Inflation is an important concept in any economic analysis because the purchasing power of money rarely stays constant. Over time, the amount of goods and services that can be purchased with a fixed amount of money tends to change (always decline). When prices inflate, we can buy less with the same amount of money.

Because of inflation, unit of currency in one period of time are not equivalent to the same unit in another time. Economic analysis requires that comparisons be made on an equivalent basis. So, it is important to incorporate the effect of inflation in analysis of alternatives. However, when the purchasing power of money increase as time passes, this is named deflation. Deflation has an opposite effect to inflation. Inflation leads to increasing the prices of projects. Let’s assume the following:

* **Inflation rate** ($f$): The inflation rate captures the decrease in the purchasing power of the currency.

* **Interest rate** ($i$): This is the interest rate measuring the real growth of money without the effect of inflation.

* **Market interest rate** ($i'$): The combined interest rate that combines both the real money growth and inflation. This is named as the inflated interest rate (interest rate considering effect of inflation).

To drive the relation between these different interest rates, when calculating the investment of a given some of money after any time, this sum of money is multiplied by $(1 + i)$ to the power of the time period. Considering the inflation effect, then the sum of money is multiplied also by $(1 + f)$. Or, this sum of money is multiplied by $(1 + i')$ which combines both the effect of interest and inflation. Accordingly, the relation could be derived as follows:

\[
(1 + i') = (1 + i)(1 + f) = 1 + i + f + if
\]

Then, \(i' = i + f + if\) \hspace{1cm} (3.6)
Usually, in the economic analysis of alternatives we consider that the inflation rate is constant all over the study period. Accordingly, in all economic analysis formulas, the normal interest rate \( i \) is replaced by the inflated interest rate \( i' \).

**Example 3.24:** What is the amount of money a company should save now to buy a new equipment costs LE65,000 after three years from now. The interest rate is 13% and the inflation rate is 7%.

**Solution:** \( i' = i + f + if = 0.13 + 0.07 + 0.13 \times 0.07 = 0.2091 = 20.91\% \)

\[
P = \frac{F}{(1 + i')^n} = \frac{65000}{(1 + 0.2091)^3}
= LE36,773
\]

**Example 3.25:** Which of the following revenues is preferred, if the interest rate is 12% and the inflation rate is 11%?

a. LE60,000 now.

b. LE16,000 annually starting one year from now for a period of 12 years.

c. LE50,000 after three years from now and LE80,000 after 5 years from now.

**Solution:** \( i' = i + f + if = 0.12 + 0.11 + 0.12 \times 0.11 = 0.243 = 24.3\% \)

\[
P_a = LE60,000
\]

\[
P_b = A\left[ ((1 + i')^n - 1) / (i'(1 + i')^n) \right]
= 16000 \left[ (1.243^{12} -1) / 0.243(1.243^{12}) \right] = LE61,003
\]

\[
P_c = 50000 / (1.243)^3 + 80000 / (1.243)^5 = LE52,996
\]

Then, option b is preferred.

**Example 3.26:** Calculate the \( NPV \) for LE35,000 invested now in addition to LE7000 annually for 5 years starting one year from now and increased by LE800 annually staring from the sixth year for the next 8 years. Interest rate is 15% and inflation rate is 10% annually.
Solution: the cash flow diagram is shown below:

![Cash Flow Diagram]

\[ i' = i + f + if = 0.15 + 0.11 + 0.15 \times 0.11 = 0.2765 = 27.65\% \]

\[ NPV = 35000 + 7000[ ((1.2765)5 - 1)) / (0.2765(1.2765)^5 ))] \]
\[ + 7800[ ((1.2765)5 - 1)) / (0.2765(1.2765)^5 ))][1/(1.2756)^5] \]
\[ + 800[ ((1.2765)^8 - 8x0.2765 -1)) / ((0.2765)^2(1.2765)^8 ))] \]
\[ [1/(1.2756)^8] \]
\[ = 35000 + 44406 + 76889 + 1681 = LE157796 \]

3.10 Exercises

1. Your company has been presented with an opportunity to invest in a project. The facts on the project are presented below:

<table>
<thead>
<tr>
<th>Investment required</th>
<th>LE60,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salvage value after 10 years</td>
<td>0</td>
</tr>
<tr>
<td>Gross income expected from the project</td>
<td>LE20,000,000/yr</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating costs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor</td>
</tr>
<tr>
<td>Materials, licenses, insurance, etc</td>
</tr>
<tr>
<td>Fuel and other costs</td>
</tr>
<tr>
<td>Maintenance costs</td>
</tr>
</tbody>
</table>

The project is expected to operate for ten years. If your management expects to make 25% on its investments before taxes, would you recommend this project?
2. A firm is trying to decide which of two alternate weighing scales to install in its plant. The scale should allow better control of the filling operation and result in loss overfilling. If both scales have lives equal to six years, which one should be selected if the interest rate is 8% assuming in the information given below.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Cost</th>
<th>Annual benefits</th>
<th>Salvage value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale A</td>
<td>LE2000</td>
<td>LE450</td>
<td>LE100</td>
</tr>
<tr>
<td>Scale B</td>
<td>LE3000</td>
<td>LE600</td>
<td>LE700</td>
</tr>
</tbody>
</table>

3. An investor paid LE8,000 to a consulting firm to analyze what he might do with a small parcel of land on the edge of town that can be bought for LE30,000. In their report, the consultants suggested four alternatives ($i = 10\%$):

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Total investment including land</th>
<th>Annual benefits</th>
<th>Terminal value at end of 20 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Do nothing</td>
<td>LE0</td>
<td>LE0</td>
<td>LE0</td>
</tr>
<tr>
<td>B: Vegetable market</td>
<td>LE50,000</td>
<td>LE5,100</td>
<td>LE30,000</td>
</tr>
<tr>
<td>C: Gas station</td>
<td>LE95,000</td>
<td>LE10,500</td>
<td>LE30,000</td>
</tr>
<tr>
<td>D: Small motel</td>
<td>LE350,000</td>
<td>LE36,000</td>
<td>LE150,000</td>
</tr>
</tbody>
</table>

4. Two pieces of construction equipment are being analyzed:

<table>
<thead>
<tr>
<th>Year</th>
<th>Alternative A</th>
<th>Alternative B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-EL2,000</td>
<td>-EL1,500</td>
</tr>
<tr>
<td>1</td>
<td>+LE1,000</td>
<td>+LE700</td>
</tr>
<tr>
<td>2</td>
<td>+LE850</td>
<td>+LE300</td>
</tr>
<tr>
<td>3</td>
<td>+LE700</td>
<td>+LE300</td>
</tr>
<tr>
<td>4</td>
<td>+LE550</td>
<td>+LE300</td>
</tr>
<tr>
<td>5</td>
<td>+LE400</td>
<td>+LE300</td>
</tr>
<tr>
<td>6</td>
<td>+LE400</td>
<td>+LE400</td>
</tr>
<tr>
<td>7</td>
<td>+LE400</td>
<td>+LE500</td>
</tr>
<tr>
<td>8</td>
<td>+LE400</td>
<td>+LE600</td>
</tr>
</tbody>
</table>

Based on an 8% interest rate, which alternative should be selected?
5. In a present worth analysis, one alternative has a net present worth of +LE420, based on a six-year analysis period that equals the useful life of the alternative. A 10% interest rate was used in the computations. The alternative is to be replaced at the end of the six years by an identical piece of equipment with the same cost, benefits and useful life. Based on a 10% rate, compute the net present worth of the equipment for the 12-year analysis period.

6. Consider five mutually exclusive alternatives:

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>LE600</td>
<td>LE600</td>
<td>LE600</td>
<td>LE600</td>
<td>LE600</td>
</tr>
<tr>
<td>Annual benefits (first 5 years)</td>
<td>LE100</td>
<td>LE100</td>
<td>LE100</td>
<td>LE150</td>
<td>LE150</td>
</tr>
<tr>
<td>Annual benefits (last 5 years)</td>
<td>LE50</td>
<td>LE100</td>
<td>LE110</td>
<td>0</td>
<td>LE50</td>
</tr>
</tbody>
</table>

The interest rate is 10%. If all alternatives have a ten-year useful life, and no salvage value, which alternative should be selected?

7. A piece of machinery was purchased for an initial cost of LE100,000. It has an anticipated life of 10 years and its salvage value is predicted to be 15% of its initial value. What must the revenues be on an annual basis to make this a worthwhile purchase? Assume an interest rate of 10%.

8. Compare between the following two alternatives presented in the following table using the EUAC method. Consider \( i = 15\% \).

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Purchase cost</th>
<th>Annual revenue</th>
<th>Salvage value</th>
<th>Useful life</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>LE11000</td>
<td>LE3500</td>
<td>LE1000</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>LE18000</td>
<td>LE3100</td>
<td>LE2000</td>
<td>9</td>
</tr>
</tbody>
</table>
9. Two machines are being considered for purchase. If the MARR is 10%, which machine should be bought?

<table>
<thead>
<tr>
<th></th>
<th>Machine X</th>
<th>Machine Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial cost</td>
<td>LE200</td>
<td>LE700</td>
</tr>
<tr>
<td>Annual benefits</td>
<td>95</td>
<td>120</td>
</tr>
<tr>
<td>Salvage value</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>Useful life, years</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

10. A new restaurant is considering buying a new meat slicer system for LE50,000 that they estimate will save them LE11,000 per year in labor and operating costs. The same system with an auto loader system is EL68,000 and will save LE14,000 per year. If the life of both systems is expected to be 8 years and their MARR is 12% which, if either system, should be accepted?

11. A company has developed a list of potential projects available for future investment. Using a MARR of 10% and using IRR method of comparison. If the projects are independent what project is recommended?

<table>
<thead>
<tr>
<th>Project</th>
<th>First Cost</th>
<th>Net Revenue (annually)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100,000</td>
<td>16,980</td>
</tr>
<tr>
<td>B</td>
<td>85,000</td>
<td>14,500</td>
</tr>
<tr>
<td>C</td>
<td>25,000</td>
<td>3,200</td>
</tr>
<tr>
<td>D</td>
<td>43,000</td>
<td>6,700</td>
</tr>
<tr>
<td>E</td>
<td>79,000</td>
<td>12,300</td>
</tr>
<tr>
<td>F</td>
<td>13,000</td>
<td>1,800</td>
</tr>
<tr>
<td>G</td>
<td>112,000</td>
<td>17,800</td>
</tr>
</tbody>
</table>

12. A new road is to be constructed between two cities. The construction cost is estimated at LE5 million, while the annual maintenance cost is LE50,000. The saving in gas consumption is LE400,000 annually. The losses from the agriculture
land used is estimated to be LE80,000 annually. Calculate the B/C ratio, if the interest rate is 5% and the road is considered as aged project. Use both the present worth and equivalent annual worth methods.

13. Consider the following six alternatives. They have 20 years useful lives and no salvage value. If the MARR is 6%, which alternative should be selected?

<table>
<thead>
<tr>
<th>Alternative</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>LE4000</td>
<td>LE2000</td>
<td>LE6000</td>
<td>LE1000</td>
<td>LE9000</td>
<td>LE10000</td>
</tr>
<tr>
<td>PW of benefits</td>
<td>LE7330</td>
<td>LE4700</td>
<td>LE8730</td>
<td>LE1340</td>
<td>LE9000</td>
<td>LE9500</td>
</tr>
</tbody>
</table>

14. A firm is purchasing production equipment for a new plant. Two alternative machines are being considered with their information as shown in the following table.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>LE30,000</td>
<td>LE35,000</td>
</tr>
<tr>
<td>Annual benefits</td>
<td>LE12,000 the first year, declining LE 3000 per year</td>
<td>LE1,000 the first year, increasing LE 3000 per year</td>
</tr>
<tr>
<td>Useful life, years</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Neither machine has any salvage value. Compute the payback period for each of the alternatives.

15. Resolve problem 14 again using the IRR method. Compare the results with that obtained from the previous problem.

16. If the initial cost for an equipment is LE18,000 and its salvage value is LE4,000. The annual revenue is LE3,500. Considering an interest rate of 12% and useful life of 10 years, what is the payback period?
17. If the initial cost of a given project is LE100,000 and the expected revenues in the next four years as follow: LE35,000, LE40,000, LE30,000 and LE25,000 respectively. If the expected inflation rate for the next four years as follows: 5%, 5%, 8% and 6% respectively. If the interest rate is 12%, calculate the NPV for this project considering the inflation and when ignoring the inflation.

18. The next table shows the information for two new pieces of equipment. Which equipment do you recommend if the interest rate is 10% and the inflation rate is 12%.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Equipment A</th>
<th>Equipment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>LE10,000</td>
<td>LE20,000</td>
</tr>
<tr>
<td>Annual maintenance</td>
<td>LE8,000</td>
<td>LE3,000</td>
</tr>
<tr>
<td>Salvage value</td>
<td>LE3,000</td>
<td>LE6,000</td>
</tr>
<tr>
<td>Useful life, years</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>