Exercise 1

1. We want to compare two sensors based upon their gain setting and detection levels. The following table provides typical membership values to represent the detection level for each of the sensors.

<table>
<thead>
<tr>
<th>Gain setting (X)</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor 1 detection level (S1)</td>
<td>0</td>
<td>0.5</td>
<td>0.65</td>
<td>0.85</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sensor 2 detection level (S2)</td>
<td>0</td>
<td>0.45</td>
<td>0.6</td>
<td>0.8</td>
<td>0.95</td>
<td>1</td>
</tr>
</tbody>
</table>

The universe of discourse is $X = \{0, 20, 40, 60, 80, 100\}$, and the membership functions for the two sensors in standard discrete form are given in the table. Find the following membership functions using standard set operations:

- a. $\mu_{S1 \cup S2}(x)$
- b. $\mu_{S1 \cap S2}(x)$
- c. $\mu_{\overline{S1}}(x)$
- d. $\mu_{\overline{S2}}(x)$
- e. $\mu_{S1}(x)$
- f. $\mu_{S2}(x)$
- g. $\mu_{\overline{S1}}(x)$
- h. $\mu_{\overline{S2}}(x)$

2. Suppose you are to address a problem in the power control of a mobile cellular phones transmitting to its base station. Let MP be the medium-power fuzzy set, and HP be the high power set. Let the universe of discourse be comprised of the discrete units of $X = \{1, 2, 3, \ldots, 10\}$. The membership functions for these two fuzzy sets are shown in the following figure. For these two fuzzy sets demonstrate union, intersection, complement, difference, De Morgan’s laws, and the excluded middle laws.

3. Using the resolution principle, decompose the following fuzzy sets using their different $\alpha$-cuts:

- a. $A = 0.1/2 + 0.4/3 + 0.5/4 + 0.6/5 + 1/7 + 1/8 + 0.5/9$
- b. $B = 0.1/2 + 0.4/(-1) + 0.1/2 + 0.9/3$
- c. $C = 0.1/0.04 + 0.3/0.08 + 0.5/0.16 + 0.7/0.32$
4. The relationship between temperature and maximum operating frequency \( R \) depends on various factors for a given electronic circuit. Let \( T \) be a temperature fuzzy set (degrees Fahrenheit) and \( F \) represent a frequency fuzzy set (in MHz) on the following universe of discourse:

\[
T = \{-100, -50, 0, 50, 100\} \quad \text{and} \quad F = \{8, 16, 25, 33\}
\]

Suppose \( R \), the relationship between \( T \) and \( F \) as follows:

\[
R = \begin{bmatrix}
-100 & -50 & 0 & 50 & 100 \\
8 & .2 & .5 & .7 & 1 & .9 \\
16 & .3 & .5 & .7 & 1 & .8 \\
25 & .4 & .6 & .8 & .9 & .4 \\
33 & .9 & 1 & .8 & .6 & .4
\end{bmatrix}
\]

The reliability of the electronic circuit is represented by the relation \( S \) between the temperature and the reliability index, \( M = \{1, 2, 4, 8, 16\} \) as follows:

\[
S = \begin{bmatrix}
1 & 2 & 4 & 8 & 16 \\
-100 & 1 & .8 & .6 & .3 & .1 \\
-50 & .7 & 1 & .7 & .5 & .4 \\
0 & .5 & .6 & 1 & .8 & .8 \\
50 & .3 & .4 & .6 & 1 & .9 \\
100 & .9 & .3 & .5 & .7 & 1
\end{bmatrix}
\]

Find a relationship between frequency and reliability index using:

a. max-min composition  

b. max-product composition

5. In the field of computer networking there is an imprecise relationship between the level of use of a network communication bandwidth and the latency experienced in peer-to-peer communications. Let \( X \) be a fuzzy set of use levels (in terms of the percentage of full bandwidth used) and \( Y \) be a fuzzy set of latencies (in milliseconds) with the following membership functions:

\[
X = \{0.2/10 + 0.5/20 + 0.8/40 + 1.0/60 + 0.6/80 + 0.1/100\} \\
Y = \{0.3/50 + 0.6/1 + 0.9/1.5 + 1.0/4 + 0.6/8 + 0.3/2\}
\]

a. Find the Cartesian product represented b the relation \( R = X \times Y \)

Now, suppose we have a second fuzzy set of bandwidth usage given by

\[
Z = \{0.3/10 + 0.6/20 + 0.7/40 + 0.9/60 + 1/80 + 0.5/100\}
\]

b. Find \( S = Z \times R \) using max-min composition.

c. Find \( S = Z \times R \) using max-product composition.