Exercise 1

1. The design engineer for a large building must determine the foundation type and size that will ensure that excessive settlement of the building does not occur. Damage costs to the building increase with the degree of settlement. The engineer has some idea of the probability of finding certain soil strength, but a better estimate can be obtained from field experiments and from laboratory tests. There are many possible experiments are available, and generally the reliability of an experimental test result increases with the cost of the test. However, for every test there is still a chance that the test result is not representative of the average soil condition. From past experience and data, the engineer can estimate the probability of having a certain soil given a particular experimental result from a particular test. The engineer wants the total expected cost of designing, construction, and maintaining the building to be a minimum including the costs of soil testing. Define:
   a. The objective function in words;
   b. Decision variables;
   c. Constraints in words; and
   d. Parameters.

2. A contractor can sell several classes of concrete at a different price per cubic meter for each class. The materials specification for each class of concrete allows the percentage by weight of cement, sand, and gravel to range between certain upper and lower bounds. The contractor knows the unit cost to the company for each of the three components of concrete, and how much of each component that is available for the company to purchase. The amount of concrete of each type that the contractor can sell is limited but known. (Hint: consider a maximization objective function, not minimization). Define:
   a. The objective function in words;
   b. Decision variables;
   c. Constraints in words; and
   d. Parameters.

3. A single reinforced rectangular concrete beam must carry a known imposed moment and shear. The span length is also known, and the deflection of the beam must not exceed a certain value. The width and depth of the beam are to be determined, as well as the area of the steel to be placed in the bottom of the beam. The cost of concrete per cubic meter and the cost of steel per kilogram are known, as well as the compressive strength of the concrete and yield strength of the steel. The designer wants to design the least-cost beam. The code for such beams states that a certain minimum amount of steel, as a percent of the total effective cross sectional area, must be present in order to avoid excessive cracking on the bottom of
the beam due to temperature fluctuations. The code also gives a limit on the maximum amount of steel, again expressed as a percent of the total effective cross sectional area of the beam, which can be present to avoid sudden compressive failure in the concrete at the top of the beam. Define:

a. The objective function in words;
b. Decision variables;
c. Constraints in words; and
d. Parameters.

4. An agency is planning a new toll exit for an existing toll highway. The number of toll booths to put at the exit is in question. The agency wants to keep costs low by having as few booths as possible. But if the waiting lines get too long during rush hours and other peak periods will hurt public comfort, reduce the number of people who will use the exit, and in the worst case, back waiting vehicles onto the highway may cause hazardous situation. The agency believes that no more than six cars, on average across the lane, should be stored in the waiting lines during rush hours, but is willing to examine other average waiting line lengths. From data elsewhere, it estimates the arrivals at the exit during each two-minute segment of the rush hour. It knows that it takes 17 seconds to service a car at a booth, resulting in 3.53 vehicles being processed each minute by a toll attendant. The agency decides to develop an optimization model to analyze the problem. The goal is to assess the impact of the number of toll booths on the average length of the waiting line. Define:

a. The objective function in words;
b. Decision variables;
c. Constraints in words; and
d. Parameters.

5. A city needs a certain amount of water from a river that is 100 m below and 5 kilometers from the city’s water treatment plant. The city engineer must decide on the internal diameter of the pipeline to the city and the head capacity of the pump to be installed at the river. The engineer knows that a smaller pipe is less expensive but results in greater head loss due to friction and therefore requires a greater head capacity for the pump, which increases the cost of the pump. Conversely, a larger pipe is more expensive, but permits a less expensive pump to meet the fixed head and flow requirement at the water treatment plant. The engineer is also aware that the range of pump capacities and pipeline diameter is limited in order to avoid a long waiting period for delivery. The most economical pump/pipe combination is desired. Define:

a. Decision variables;
b. Parameters;
c. The objective function in words; and
d. Constraints in words.