Exercise 4

1. Transform the following linear program model into standard form for the simplex method

   \[ \text{Minimize} \quad -2x_1 + 3x_2 \]
   \[ x_1 - 3x_2 + 2x_3 \leq 3 \]
   \[ -x_1 + 2x_2 \geq 2 \]
   \[ 2x_2 \leq -5 \]
   \[ x_1 \geq 0; \ x_2 \geq 0; \ \text{and} \ x_3 \text{ unrestricted} \]

2. Solve analytically and verify graphically, the following LP model:

   \[ \text{Maximize} \quad z = x_1 + x_2 \]
   \[ \text{Subject to} \]
   \[ x_1 - 2x_2 \leq 4 \]
   \[ 2x_1 + 3x_2 \leq 12 \]
   \[ 3x_1 + 4x_2 \leq 12 \]
   \[ x_1 \geq 0; \ x_2 \geq 0 \]

3. Use the simplex method to solve the problem:

   \[ \text{Maximize} \quad z = 3x_1 + 2x_2 \]
   \[ \text{Subject to} \]
   \[ x_1 + 2x_2 \leq 6 \]
   \[ 2x_1 + x_2 \leq 8 \]
   \[ -x_1 + x_2 \leq 1 \]
   \[ x_2 \leq 2 \]
   \[ x_1 \geq 0; \ x_2 \geq 0 \]

   a. Determine the basic feasible solution at each iteration.
   b. Define the basic and nonbasic variables at the optimal solution.
   c. Determine the maximum value of \( z \).

4. Use the simplex method to solve the problem:

   \[ \text{Maximize} \quad z = x_1 + 2x_2 + 3x_3 \]
   \[ \text{Subject to} \]
   \[ 2x_1 + x_2 + x_3 \leq 4 \]
   \[ x_1 + 2x_2 + x_3 \leq 4 \]
   \[ x_1 + x_2 + 2x_3 \leq 4 \]
   \[ x_1 + x_2 + x_3 \leq 3 \]
   \[ x_1 \geq 0; \ x_2 \geq 0; \ x_3 \geq 0 \]
d. Determine the basic feasible solution at each iteration.
e. Define the basic and nonbasic variables at the optimal solution.
f. Determine the maximum value of z.

5. Use the simplex method to solve the problem:

Maximize \( z = x_1 + 2x_2 + 4x_3 \)
Subject to
\[
\begin{align*}
3x_1 + x_2 + 5x_3 &\leq 10 \\
x_1 + 4x_2 + x_3 &\leq 8 \\
2x_1 + 2x_3 &\leq 7 \\
x_1 + x_2 + x_3 &\leq 3 \\
x_1 &\geq 0; \; x_2 &\geq 0; \; x_3 &\geq 0
\end{align*}
\]

g. Determine the basic feasible solution at each iteration.
h. Define the basic and nonbasic variables at the optimal solution.
i. Determine the maximum value of z.

6. Consider the linear program

Maximize \( 5x_1 + 3x_2 + x_3 \)
Subject to
\[
\begin{align*}
x_1 + x_2 + x_3 &\leq 6 \\
5x_1 + 3x_2 + 6x_3 &\leq 15 \\
x_1; \; x_2; \; x_3 &\geq 0
\end{align*}
\]

and an associated tableau

<table>
<thead>
<tr>
<th></th>
<th>( z )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>RHS</th>
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<td>5</td>
<td>0</td>
<td>1</td>
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<td></td>
</tr>
<tr>
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<td>-0.2</td>
<td>1</td>
<td>-0.2</td>
<td>3</td>
<td></td>
</tr>
<tr>
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<td>1</td>
<td>0.6</td>
<td>1.2</td>
<td>0</td>
<td>0.2</td>
<td>3</td>
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</tr>
</tbody>
</table>

(a) What basic solution does this tableau represent? Is this solution optimal? Why or why not?
(b) Does this tableau represent a unique optimum? If not, find an alternative optimal solution.

7. Solve the following linear program using the simplex method. Compute the value of the objective function and decision variables at optimality, and indicate which statement best describes the solution and why: a) this linear program has a unique optimal solution; b) this linear program has alternate optima; c) this linear program is infeasible; or d) this linear program is unbounded.

Minimize \( z = 2x_1 + 3x_2 + x_3 \)
Subject to
\[
\begin{align*}
2x_1 + x_2 - x_3 &\geq 3 \\
x_1 + x_2 + x_3 &\geq 2 \\
x_1; \; x_2; \; x_3 &\geq 0
\end{align*}
\]